DESCRIPTIONS OF COMPREHENDING OF FUNCTION CONCEPTS BASED ON APOS THEORY ON SEMESTER III STUDENTS IN MATHEMATICS EDUCATION STUDY PROGRAM FACULTY OF TARBIYAH AND EDUCATION UIN ALAUDDIN MAKASSAR

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ABSTRACT

This study aimed to obtain descriptions of the concept of function based on APOS Theory. This research was a qualitative descriptive study with 3 students, as research subjects, of Mathematics Education Study Program Faculty of Tarbiyah and Education UIN Alauddin Makassar Academic Year 2017/2018 which were selected based on the Grade Point Average and predetermined criteria. The main instrument of this study was the researcher himself with two auxiliary instruments, namely tests of conceptual comprehending of functions based on APOS theory and interview guidelines. Checking the validity of data, in this study, by using time triangulation. The results of study showed that: (1) high category subject was in re-explaining the notion of concept of function based on what had been studied previously at the stage of understanding action, explaining the concept of function by using their own words or in the form of mathematical symbols at the process of understanding the process, giving examples and non examples of function concepts and their reasons were at the stage of understanding objects, and linking the relationship between a concept and other concepts was at the stage of understanding the scheme except on the concept of function composition; (2) Medium category subject was in re-explaining the notion of concept of function based on what had been studied before at the stage of understanding action, explaining the notion of the function by using their own words or in the form of mathematical symbols at the stage of understanding processes, giving examples and non-examples the concept of function and their reasons were at the stage of understanding the object except for the concept of the types of functions according to their nature and the concept of inverse functions. Medium category subjects were not able to link the relationship between a concept and other concepts; (3) Low category subject was in re-explaining the notion of the function concept based on what had been studied before at the stage of understanding action except on the concept of the types of functions according to their nature. Low category subjects were not able to explain the notion of the concept of function by using their own words or in the form of mathematical symbols, giving examples and non examples of functional concepts and their reasons, and linking the relationship between a concept and other concepts.

Keywords: Comprehending of Concepts, APOS Theory, Function.

PRELIMINARY

Mathematics is one of the most important subjects in the success of educational programs because it is a part of academic education and a basic science for other disciplines, also as means for students to be able to think logically, critically and systematically. The role of mathematics, hence, is so important, students are required to be able to master mathematical concepts as early as possible completely.

Conceptual comprehending is a key aspect of learning (Santrock, 2015). The definition of conceptual comprehending according to (Kilpatrick, Swafford, Findell, National Research Council (U.S.), & Mathematics Learning Study Committee, 2001) is the ability of students to understand the concepts, operations and relationships in mathematics. The indicators of conceptual understanding according to the Ministry of Education and Culture (2014) are; restating the concept that has been studied, classifying the objects based on whether or not they fulfill the requirements that form the concept, identifying the properties of operations or concepts, applying concepts logically, providing examples or not examples of concepts learned, presenting the concepts in various forms of representation mathematically (tables, graphs, diagrams, drawings, sketches, mathematical models, or other means), linking various concepts in mathematics as well as outside mathematics, developing necessary conditions and / or enough requirements of a concept.

Function is one of the important materials that must be learned in mathematics. The concept of this abstract function will prepare students in calculus lectures by examining basic ideas about functions, graphics and how to transform and combine a function. Students' understanding of the concept of function can be analyzed through an analysis of genetic decomposition as an operationalization of APOS Theory (Action, Process, Object, and Schema). APOS theory is a construction about how the possibility of achievement / learning of a mathematical concept or principle, which is used as an elaboration of mental constructs of actions, processes, objects, and schemes (Dubinsky, 2000).

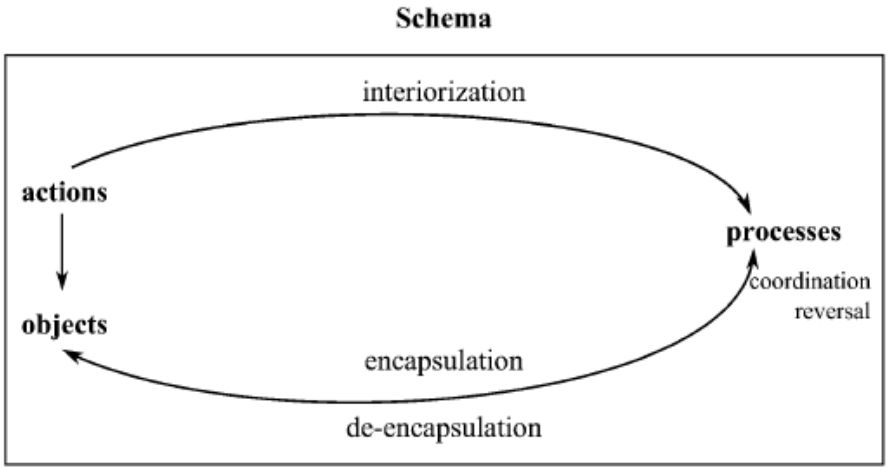

Figure 1 Structure and mechanism of mental in the construction of mathematical knowledge (Arnon et al., 2014)

Asiala et al., (1996) explained that understanding a mathematical concept begins with manipulating mental or physical objects that has previously been constructed. This manipulation forms action. The action is, then, interiorized to form the process. The process is formed because of the repetition of action and reflection on the actions taken. The process is, then, encapsulated into objects. Object means a concept has been formed in the mind of an individual. The object itself can be de-encapsulated again into a process. Then, in the end the actions, processes, and objects together with other schemes are organized into a complete scheme of concepts and can be used to solve mathematical problems.

Based on the description of the problem above, the author is interested in conducting research with the topic "Descriptions of comprehending of function concept based on APOS Theory on Semester III students of Tarbiyah and Education Faculty Mathematics Education Program at UIN Alauddin Makassar?"

The purpose of this study is to obtain descriptions of the comprehending of function concept based on APOS Theory on Semester III students of Mathematics Education Study Program at Tarbiyah and Education Faculty of UIN Alauddin Makassar.

METHOD

This type of research was a descriptive study by using a qualitative approach. The subjects of this study were the third semester students of Mathematics Education Study Program Faculty of Tarbiyah and Education UIN Alauddin Makassar Academic Year 2017/2018. In order to determine research subjects who had predetermined criteria, the researcher grouped the research subjects into 3 (three) categories based on GPA obtained from Semester I and II, namely group with high GPA category, group with medium GPA category, and group with Low GPA category. As shown in the following table:

Table 1 Achievement Index

Achievement Index	Category
3,51 – 4,00	High
3,01 – 3,50	Medium
2,76 – 3,00	Low

Source: Education Guidelines of UIN Alauddin Makassar, 2016

The research instrument used tests of comprehending of unction concept and interviews that were previously validated by 2 validators. The collected data ws in the form of answers to the results of functional concept comprehending tests based on APOS theory and interview transcripts analyzed by using data analysis techniques, namely data reduction, data display (data presentation), and conclusion drawing / verification (conclusion drawing / verification).

RESULTS AND DISCUSSION

The results of the completion of written test and interview were used to determine the level of comprehending of function concept based on APOS theory of three research subjects. The following is a description of comprehending of function concepts based on APOS theory in each subject of the study to be discussed as follows:

1. Achieving the level of comprehending of function concepts based on APOS Theory with High Category Subject

Tabel 2 Achieving the level of comprehending of function concepts based on APOS Theory with high category subject

No.	Concept of Function Definition	Stage of APOS Theory			
		A	P	O	S
1.	Re-explaining the meaning of function based on what have been learned before	✓	-	-	-
2.	Explaining the meaning of function by using their own words or in the form of mathematical symbols	-	✓	-	-
3.	Giving examples and non examples of functions and their reasons	-	-	✓	-
4.	Explaining the relationship between the concept of function and other concepts	-	-	-	✓

Table 2 showed that KT subject in explaining the meaning of functions based on what has been studied previously at the stage of action, describing the meaning of the function by using their own words or in the form of mathematical symbols at the process stage, giving examples and non examples of functions and their reasons at the object stage, and explaining the relationship between the concept of function and other concepts at the schematic stage.

Table 3 Achievement the level of comprehending of types of functions concept based on APOS Theory with High Category Subjects

No.	Types of Function Concept	Stage of APOS Theory			
		A	P	O	S
1.	Re-explaining the meaning of injective, objective, and wise functions based on what has been learned before	✓	-	-	-
2.	Explaining the meaning of injective, objective, and wise functions by using their own words or in the form of mathematical symbols	-	✓	-	-

3.	Giving examples and non examples of injective, objective, and wise functions and their reasons	-	-	✓	-
4.	Explaining the relationship between the concept of a functional function and other concepts	-	-	-	✓

Table 3 showed that KT subject in explaining the comprehending of types of functions based on what has been learned before at the stage of action, explaining the comprehending of the types of functions by using their own words or in the form of mathematical symbols at the process stage, giving an explanation of the functions provided include injective, objective, and wise functions are at the object stage, and explaining the relationship between the concept of the wise function and the concepts of other types of functions are at the schema stage.

Table 4 Achievement the level of comprehending of the concept of function composition based on APOS Theory with High Category Subject

No.	**Concept of function Composition**	**Stage of APOS Theory**			
		A	**P**	**O**	**S**
1.	Re-explaining the notion of function composition based on what has been studied before	✓	-	-	-
2.	Explaining the meaning of the composition of functions by using their own words or in the form of mathematical symbols	-	✓	-	-
3.	Giving examples and non examples of the composition of functions and their reasons	-	-	✓	-
4.	Explaining the relationship between the concept of function composition and other concepts	-	-	-	-

Table 4 showed that KT subject in explaining the notion of function composition based on what has been studied previously at the action stage, explaining the meaning of function composition by using their own words or in the form of mathematical symbols at the process stage, giving examples of two functions that can be composed

and examples of functions that cannot be composed and the reasons are at the object stage. Subject KT was unable to explain the relationship between the concept of function composition and other concepts.

Table 5 Achievement the level of comprehding of the concept of inverse functions based on APOS Theory with High Category Subjects

No.	Concept of Inverse Function	Stage of APOS Theory			
		A	P	O	S
1.	Re-explaining the definition of inverse function based on what has been learned before	✓	-	-	-
2.	Explaining the meaning of inverse functions by using their own words or in the form of mathematical symbols	-	✓	-	-
3.	Giving examples and non examples of inverse functions and their reasons	-	-	✓	-
4.	Explaining the relationship between inverse function concepts and other concepts	-	-	-	✓

Table 5 showed that KT subject in explaining the definition of inverse function based on what has been studied before at the action stage, explaining the meaning of inverse function by using their own words or in the form of mathematical symbols at the process stage, giving examples and non examples of inverse functions along with the reasons for being at the object stage, and explaining the relationship between the concept of function and other concepts at the schematic stage.

2. Achievement the level of Comprehending of function concepts based on APOS Theory with Medium Category Subject

Table 6 Achievement the level of comprehending of the concept of function definition based on APOS Theory with Medium Category Subject.

No.	Concept of Function Definition	Stage of APOS Theory			
		A	P	O	S
1.	Re-explaining the meaning of functions based on what has been learned before	✓	-	-	-

2.	Explaining the meaning of functions by using their own words or in the form of mathematical symbols	-	✓	-	-
3.	Giving examples and non examples of functions and their reasons	-	-	✓	-
4.	Explaining the relationship between the concept of function and other concepts	-	-	-	-

Table 6 showed that subject of KS in explaining the meaning of functions based on what has been studied before at the stage of action, explaining the meaning of functions by using their own words at the process stage, and giving examples and non examples of functions and reasons at the object stage. KS subjects cannot explain the relationship between the concept of function and other concepts.

Table 7 Achievement the level of comprehending of types of functions concept based on APOS Theory with Medium Category Subject

No.	**Types of Function Concept**	**Stage of APOS Theory**			
		A	**P**	**O**	**S**
1.	Re-explaining the meaning of injective, objective, and wise functions based on what has been learned before	✓	-	-	-
2.	Explaining the meaning of injective, objective, and wise functions by using their own words or in the form of mathematical symbols matematis	-	✓	-	-
3.	Giving examples and non examples of functions and their reasons	-	-	-	-
4.	Explaining the relationship between the concept of a functional function and other concepts	-	-	-	-

Table 7 showed that subject of KS in explaining the comprehending of the types of functions based on what has been studied before at the stage of action, explaining the meaning of types of functions by using their own words or in the form of mathematical symbols at the process stage. The KS subject is unable to provide an

explanation of the functions given including injective, objective, and wise functions and is not able to explain the relationship between the concept of the functional function and other types of function concepts.

Table 8 Achievement the level of comprehending of the concept of function composition based on APOS Theory with Medium Category Subject

No.	Consept of Function Composition	Stage of APOS Theory			
		A	P	O	S
1.	Re-explaining the notion of function composition based on what has been studied before	✓	-	-	-
2.	Explaining the meaning of the composition of functions by using their own words or in the form of mathematical symbols	-	✓	-	-
3.	Giving examples and non examples of the composition of functions and their reasons	-	-	✓	-
4.	Explaining the relationship between the concept of function composition and other concepts	-	-	-	-

Table 8 showed that subject of KS in re-explaining the notion of function composition based on what has been studied previously at the stage of action, explaining the meaning of function composition by using their own words or in the form of mathematical symbols at the process stage, and giving examples of two functions that can composed and examples of functions that cannot be composed and the reasons are at the object stage. The KS subject is unable to explain the relationship between the concept of function composition and other concepts.

Table 9 Achievement the level of comprehending of the concept of inverse function based on APOS Theory with Medium Category Subject

No.	Concept of Inverse Function	Stage of APOS Theory			
		A	P	O	S
1.	Re-explaining the definition of inverse function based on what has been learned before	✓	-	-	-

2.	Explaining the meaning of inverse functions by using their own words or in the form of mathematical symbols	✓	-	-	-
3.	Giving examples and non examples of inverse functions and their reasons	-	-	-	-
4.	Explaining the relationship between inverse function concepts and other concepts	-	-	-	-

Table 9 showed that KS subject in explaining the definition of inverse function based on what has been studied before is at the action stage. KS subject is less able to explain the meaning of inverse function by using their own words. KS subjects are not able to provide examples and non examples of inverse functions and their reasons and are unable to explain the relationship between the concept of function and other concepts.

3. Achieviement the level of comprehending of function concepts based on APOS Theory with Low Category Subject

Table 10 Achievement the level of comprehending of the concept of function definition based on APOS Theory with Low Category Subject

No.	**Concept of Function Definition**	**Stage of APOS Theory**			
		A	**P**	**O**	**S**
1.	Re-explaining the meaning of functions based on what has been learned before	✓	-	-	-
2.	Explaining the meaning of functions by using their own words or in the form of mathematical symbols	-	-	-	-
3.	Giving examples and non examples of functions and their reasons	-	-	-	-
4.	Explaining the relationship between the concept of function and other concepts	-	-	-	-

Table 10 showed that KR subject in explaining the meaning of functions based on what has been studied previously is at the stage of action. Subject KR is not able to explain the meaning of a function by using their own words. KR subject is not able to

provide examples and non examples of functions and their reasons. Subject KR is unable to explain the relationship between the concept of function and other concepts.

Table 11 Achievement the level of comprehending of types of functions concept based on APOS Theory with Low Category Subject

No.	Types of Function Concept	Stage of APOS Theory			
		A	P	O	S
1.	Re-explaining the meaning of injective, objective, and wise functions based on what has been learned before	-	-	-	-
2.	Explaining the meaning of injective, objective, and wise functions by using their own words or in the form of mathematical symbols	-	-	-	-
3.	Giving examples and non examples of types of functions according to their nature and reasons	-	-	-	-
4.	Explaining the relationship between the concept of function composition and other concepts	-	-	-	-

Table 11 showed that KR subject is unable to re-explain the comprehending of the types of functions based on what has been studied before. The KR subject is not able to explain the types of functions by using their own words or in the form of mathematical symbols. The KR subject is unable to provide an explanation of the functions given including injective, objective, and wise functions. Subject KR is not able to explain the relationship between the concept of wise function and other types of function concepts.

Table 12 Achievement the level of comprehending of the concept of function composition based on APOS Theory with Low Category Subject

No.	Consept of Function Composition	Stage of APOS Theory			
		A	P	O	S
1.	Re-explaining the notion of function composition based on what has been studied before	✓	-	-	-

2.	Explaining the meaning of the composition of functions by using their own words or in the form of mathematical symbols	-	✓	-	-
3.	Giving examples and non examples of the composition of functions and their reasons	-	-	-	-
4.	Explaining the relationship between the concept of function composition and other concepts	-	-	-	-

Table 12 explained that KR subject in re-explaining the notion of function composition is based on what has been studied before. Subject KR is unable to explain the definition of function composition using his own words or in the form of mathematical symbols. KR subject is not able to provide examples of two functions that can be composed and examples of functions that cannot be compiled and why. Subject KR is not able to explain the relationship between the concept of function composition and other concepts.

Table 13 Achievement the level of comprehending of the concept of inverse functions based on APOS Theory with Low Category Subject

No.	**Concept of Inverse Function**	**Stage of APOS Theory**			
		A	**P**	**O**	**S**
1.	Re-explaining the definition of inverse function based on what has been learned before	✓	-	-	-
2.	Explaining the meaning of inverse functions by using their own words or in the form of mathematical	-	-	-	-
3.	Giving examples and non examples of inverse functions and their	-	-	-	-
4.	Explaining the relationship between inverse function concepts and other concepts	-	-	-	-

Table 13 showed that KR subject in explaining the definition of inverse function based on what has been studied previously is at the action stage. Subject KR is not able to explain the meaning of inverse function by using his own words. KR subject

is not able to provide examples and non examples of inverse functions and their reasons. Subject KR is unable to explain the relationship between the concept of function and other concepts.

CONCLUSION

Based on the results and analysis of research data and discussion, so the conclusions as follows:

1. Subject of KT was able to re-explain the notion of function based on what had been studied before, re-explain the understanding of the types of functions according to their characteristics based on what had been studied before, re-explain the notion of function composition based on what had been studied before, and explain again the notion of inverse function based on what had been learned before. Based on these matters, it could be concluded that the subject of KT was at the level of comprehending of action. Subject KT was able to explain the meaning of function by using their own words or in the form of mathematical symbols, explaining the meaning of injective, objective, and wise functions by using their own words or in the form of mathematical symbols, explaining the meaning of function composition based on what had been studied before, and explain the meaning of inverse functions based on what had been studied before. Based on these matters, it could be concluded that the KT subject was at the level of understanding the process. Subject KT was able to provide and explain examples and non examples of functions and their reasons, explain the examples given whether including injective, objective, and wise functions, give and explain examples and non-examples of function compositions, give and explain examples and non-examples of inverse functions. Based on these matters, it could be concluded that the subject KT ws at the level of understanding the object. Subjek KT was able to explain the relationship between the concept of function definition and other concepts, explaining the relationship between the concept of a functional function and other concepts, explaining the relationship between the

concept of inverse function and other concepts. Based on these matters, it could be concluded that the KT subject was at the level of understanding of the scheme.

2. KS subject was able to re-explain the meaning of the function based on what had been studied before, explain again the understanding of the types of functions according to their properties based on what had been studied before, re-explain the notion of function composition based on what had been previously studied, explain the understanding of functions inverses based on what had been studied before, explain the meaning of inverse functions by using their own words or in the form of mathematical symbols Based on these things, it could be concluded that the subject of KS was at the level of understanding action. The KS subject was able to explain the meaning of the function by using their own words or in the form of mathematical symbols, explaining the meaning of injective, objective, and wise functions by using their own words or mathematical symbols, explaining the meaning of function composition by using their own words or in the form of mathematical symbols. Based on these matters, it could be concluded that the subject of KS was at the level of understanding the process. The KS subject was able to provide and explain examples and non examples of functions and their reasons, provide and explain examples and non examples of function compositions. Based on these matters, it could be concluded that the subject of KS was at the level of understanding the object.
3. Subject of KR was able to re-explain the notion of functions based on what had been studied before, re-explain the notion of function composition based on what had been studied before, and re-explain the notion of inverse functions based on what had been studied before. Based on these matters, it could be concluded that the subject of KS was at the level of understanding the action.

REFERENCES

Arnon, I., Cottrill, J., Dubinsky, E., Oktaç, A., Roa Fuentes, S., Trigueros, M., & Weller, K. (2014). *APOS Theory*. New York, NY: Springer New York. https://doi.org/10.1007/978-1-4614-7966-6

Asiala, M., Brown, A., DeVries, D., Dubinsky, E., Mathews, D., & Thomas, K. (1996). A framework for research and curriculum development in undergraduate mathematics education. In J. Kaput, A. Schoenfeld, E. Dubinsky, & T. Dick (Eds.), *CBMS Issues in Mathematics Education* (Vol. 6, pp. 1–32). Providence, Rhode Island: American Mathematical Society. https://doi.org/10.1090/cbmath/006/01

Depdikbud. (2014). *Peraturan Pemerintah No. 59 tentang Kurikulum SMA*. Jakarta: Kementerian Pendidikan dan Kebudayaan.

Dubinsky, E. (2000). Using a theory of learning in college mathematics courses. *Teaching and Learning Undergraduate Mathematics*, *12*.

Kilpatrick, J., Swafford, J., Findell, B., National Research Council (U.S.), & Mathematics Learning Study Committee. (2001). *Adding it up: helping children learn mathematics*. Washington, DC: National Academy Press. Retrieved from http://site.ebrary.com/id/10038695

Santrock, J. W. (2015). *Psikologi Pendidikan* (Edisi Kedua). Jakarta: Kencana.